

**“JUST THE MATHS”**

**SLIDES NUMBER**

**8.2**

**VECTORS 2**

**(Vectors in component form)**

**by**

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**8.2.1 The components of a vector**

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**8.2.3 The sum and difference of vectors in component form**

**8.2.4 The direction cosines of a vector**

## UNIT 8.2 - VECTORS 2

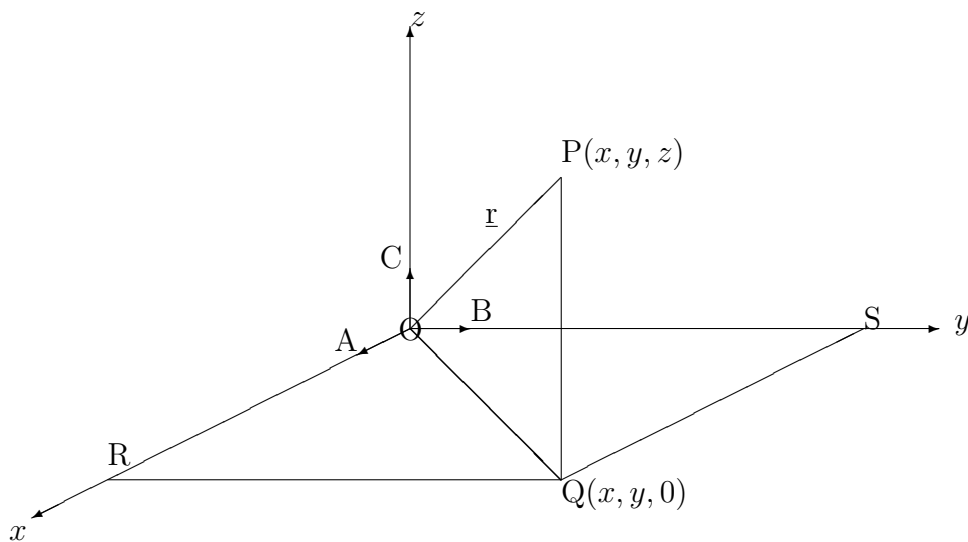
### VECTORS IN COMPONENT FORM

#### 8.2.1 THE COMPONENTS OF A VECTOR

Vectors in space are defined in terms of **unit vectors** placed along the axes  $Ox$ ,  $Oy$  and  $Oz$  of a three-dimensional right-handed cartesian reference system.

These unit vectors will be denoted respectively by  **$\mathbf{i}$** ,  **$\mathbf{j}$** , and  **$\mathbf{k}$** ; (“bars” and “hats” may be omitted).

Consider the following diagram:



In the diagram,  $\underline{OA} = \mathbf{i}$ ,  $\underline{OB} = \mathbf{j}$  and  $\underline{OC} = \mathbf{k}$ .

P is the point with co-ordinates  $(x, y, z)$ .

By the Triangle Law,

$$\underline{r} = \underline{OP} = \underline{OQ} + \underline{QP} = \underline{OR} + \underline{RQ} + \underline{QP}.$$

That is,

$$\underline{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

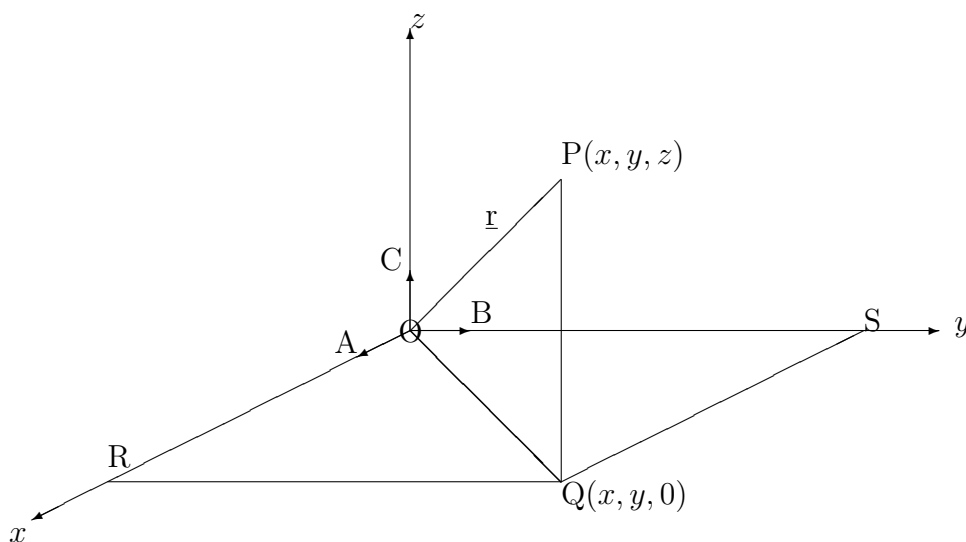
**Note:**

Vectors which emanate from the origin are not a special case since we are dealing with free vectors.

Nevertheless,  $\underline{OP}$  is called the position vector of the point P.

The numbers  $x$ ,  $y$  and  $z$  are called the “**components**” of  $\underline{OP}$  (or of any other vector in space with the same magnitude and direction as  $\underline{OP}$ ).

## 8.2.2 THE MAGNITUDE OF A VECTOR IN COMPONENT FORM



By Pythagoras' Theorem,

$$(OP)^2 = (OQ)^2 + (QP)^2 = (OR)^2 + (RQ)^2 + (QP)^2$$

That is,

$$r = |x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}.$$

### EXAMPLE

Determine the magnitude of the vector

$$\underline{\mathbf{a}} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

and hence obtain a unit vector in the same direction.

## Solution

$$|\underline{a}| = a = \sqrt{5^2 + (-2)^2 + 1^2} = \sqrt{30}.$$

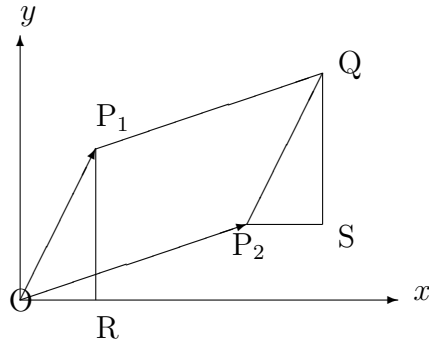
A unit vector in the same direction as  $\underline{a}$  is obtained by normalising  $\underline{a}$ , that is, dividing it by its own magnitude.

The required unit vector is

$$\hat{\underline{a}} = \frac{1}{a} \cdot \underline{a} = \frac{5\mathbf{i} - 2\mathbf{j} + \mathbf{k}}{\sqrt{30}}.$$

### 8.2.3 THE SUM AND DIFFERENCE OF VECTORS IN COMPONENT FORM

Consider, first, a situation in **two** dimensions:



In the diagram, suppose  $P_1$  has co-ordinates  $(x_1, y_1)$  and suppose  $P_2$  has co-ordinates  $(x_2, y_2)$ .

$\Delta^{gl} ORP_1$  has exactly the same shape as  $\Delta^{gl} P_2SQ$ .

Hence, the co-ordinates of  $Q$  must be  $(x_1 + x_2, y_1 + y_2)$ .

By the Parallelogram Law,  $\underline{OQ}$  is the sum of  $\underline{OP_1}$  and  $\underline{OP_2}$ .

That is,

$$(x_1\mathbf{i} + y_1\mathbf{j}) + (x_2\mathbf{i} + y_2\mathbf{j}) = (x_1 + x_2)\mathbf{i} + (y_1 + y_2)\mathbf{j}.$$

It can be shown that this result applies in three dimensions also.

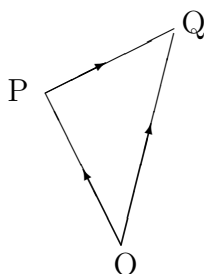
To find the **difference** of two vectors, we calculate the difference of their separate components.

## EXAMPLE

Two points P and Q in space have cartesian co-ordinates  $(-3, 1, 4)$  and  $(2, -2, 5)$  respectively.

Determine the vector  $\underline{PQ}$ .

## Solution



$$\underline{OP} = -3\mathbf{i} + \mathbf{j} + 4\mathbf{k}.$$

$$\underline{OQ} = 2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}.$$

By the triangle Law,

$$\underline{PQ} = \underline{OQ} - \underline{OP} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}.$$

## Note:

The vector drawn from the origin to the point  $(5, -3, 1)$  is the **same** as the vector  $\underline{PQ}$ .

## 8.2.4 THE DIRECTION COSINES OF A VECTOR

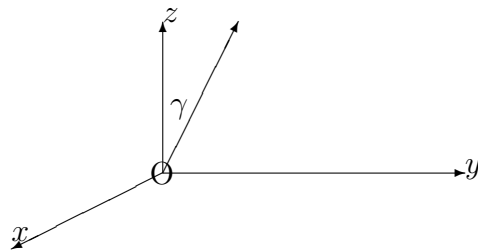
Suppose that

$$\underline{OP} = \underline{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

and suppose that  $\underline{OP}$  makes angles  $\alpha$ ,  $\beta$  and  $\gamma$  with  $Ox$ ,  $Oy$  and  $Oz$  respectively.

Then

$$\cos \alpha = \frac{x}{r}, \quad \cos \beta = \frac{y}{r} \quad \text{and} \quad \cos \gamma = \frac{z}{r}.$$



$\cos \alpha$ ,  $\cos \beta$  and  $\cos \gamma$  are called the “**direction cosines**” of  $\underline{r}$ .

Any three numbers in the same ratio as the direction cosines are said to form a set of “**direction ratios**” for the vector  $\underline{r}$ .

$x : y : z$  is one possible set of direction ratios.

## EXAMPLE

The direction cosines of the vector

$$6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

are

$$\frac{6}{\sqrt{41}}, \frac{2}{\sqrt{41}} \text{ and } \frac{-1}{\sqrt{41}}$$

since the vector has magnitude  $\sqrt{36 + 4 + 1} = \sqrt{41}$ .

A set of direction ratios for this vector are  $6 : 2 : -1$ .