

**“JUST THE MATHS”**

**SLIDES NUMBER**

**5.3**

**GEOMETRY 3**  
**(Straight line laws)**

by

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**5.3.1 Introduction**

**5.3.2 Laws reducible to linear form**

**5.3.3 The use of logarithmic graph paper**

## UNIT 5.3 GEOMETRY 3

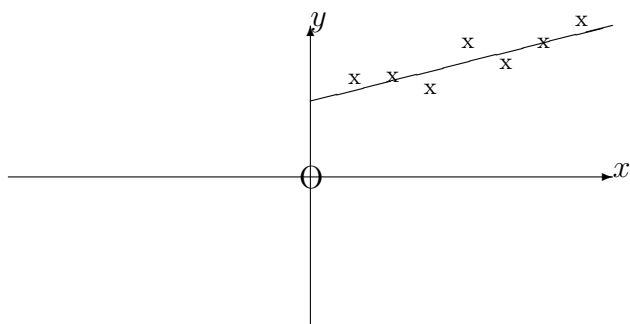
### STRAIGHT LINE LAWS

#### 5.3.1 INTRODUCTION

Suppose that two variables  $x$  and  $y$  are connected by a “**straight line law**”,

$$y = mx + c.$$

To estimate  $m$  and  $c$ , we could plot a graph of  $y$  against  $x$  and obtain the “**best straight line**” passing through (or near) the plotted points.



The “best straight line” averages out experimental errors.

Points which are out of character with the rest are usually ignored.

Measuring the gradient,  $m$ , and the intercept,  $c$ , on the  $y$ -axis is **not** always the wisest way of proceeding and should be avoided.

The reasons are as follows:

- (i) The intercept may be “off the page”.
- (ii) Symbols other than  $x$  or  $y$  may leave doubts as to which is the equivalent of the  $y$ -axis and which is the equivalent of the  $x$ -axis.

### **Standard Method**

Take two sets of readings,  $(x_1, y_1)$  and  $(x_2, y_2)$ , from the best straight line drawn; then solve the simultaneous linear equations

$$\begin{aligned}y_1 &= mx_1 + c, \\y_2 &= mx_2 + c.\end{aligned}$$

Choosing the two points as far apart as possible will reduce errors in calculation due to the use of small quantities.

## 5.3.2 LAWS REDUCIBLE TO LINEAR FORM

Experimental laws which are not linear can sometimes be reduced to linear form.

### EXAMPLES

1.

$$y = ax^2 + b.$$

#### Method:

Let  $X = x^2$  so that  $y = aX + b$ .

We may obtain a straight line by plotting  $y$  against  $X$ .

2.

$$y = ax^2 + bx.$$

#### Method:

Since

$$\frac{y}{x} = ax + b,$$

we may let  $Y = \frac{y}{x}$ , giving  $Y = ax + b$ .

A straight line will be obtained if we plot  $Y$  against  $x$ .

#### Note:

If one of the sets of readings is  $(x, y) = (0, 0)$ , we must ignore it in this example.

3.

$$xy = ax + b.$$

**Methods:**

(a) Letting  $xy = Y$  gives  $Y = ax + b$ , and we could plot  $Y$  against  $x$ .

(b) Writing  $y = a + \frac{b}{x}$ , we could let  $\frac{1}{x} = X$ , giving  $y = a + bX$ .

In this case, a straight line is obtained if we plot  $y$  against  $X$ .

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4.

$$y = ax^b.$$

**Method:**

Taking logarithms of both sides (base 10 will do here),

$$\log_{10} y = \log_{10} a + b \log_{10} x.$$

Letting  $\log_{10} y = Y$  and  $\log_{10} x = X$  gives

$$Y = \log_{10} a + bX.$$

A straight line will be obtained by plotting  $Y$  against  $X$ .

5.

$$y = ab^x.$$

**Method:**

Here again, logarithms may be used to give

$$\log_{10} y = \log_{10} a + x \log_{10} b.$$

Letting  $\log_{10} y = Y$ , we have

$$Y = \log_{10} a + x \log_{10} b.$$

A straight line is obtained if we plot  $Y$  against  $x$ .

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6.

$$y = ae^{bx}.$$

**Method:**

Taking **natural** logarithms of both sides,

$$\log_e y = \log_e a + bx,$$

also written

$$\ln y = \ln a + bx.$$

Letting  $\ln y = Y$ , we obtain a straight line by plotting  $Y$  against  $x$ .

### 5.3.3 THE USE OF LOGARITHMIC GRAPH PAPER

In logarithmic examples, we may use a special kind of graph paper on which there is printed a logarithmic scale along one or both of the axis directions.

0.1    0.2    0.3    0.4            1    2    3    4            10

A logarithmic scale evaluates the logarithms of the numbers assigned to it provided these numbers are allocated to each “**cycle**” of the scale in successive powers of 10.

Data which includes numbers spread over several different successive powers of ten will need graph paper which has at least that number of cycles in the appropriate axis direction.

#### EXAMPLE

The numbers 0.03, 0.09, 0.17, 0.33, 1.82, 4.65, 12, 16, 20, 50 will need **four** cycles on a logarithmic scale.

These restrictions make logarithmic graph paper less economical to use than ordinary graph paper.

On a logarithmic scale, we plot the **actual** values of the variables whose logarithms we would otherwise have needed to look up.

This will give the straight line graph from which we take the usual two sets of readings.

The two sets of readings are substituted into the form of the experimental equation which occurs immediately after taking logarithms of both sides.

Any base of logarithms may be used since logarithms to two different bases are proportional to each other.

## **EXAMPLES**

1.

$$y = ax^b.$$

### **Method**

(i) Taking logarithms (to base 10) of both sides,

$$\log_{10} y = \log_{10} a + b \log_{10} x.$$

(ii) Plot a graph of  $y$  against  $x$ , both on logarithmic scales.

(iii) Estimate the “best straight line”.

(iv) Read off from the graph two sets of co-ordinates,  $(x_1, y_1)$  and  $(x_2, y_2)$ , as far apart as possible.

(v) Solve for  $a$  and  $b$  the simultaneous equations,

$$\log_{10} y_1 = \log_{10} a + b \log_{10} x_1,$$

$$\log_{10} y_2 = \log_{10} a + b \log_{10} x_2.$$

If it is possible to choose readings which are powers of 10, so much the better, but this is not essential.

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2.

$$y = ab^x.$$

### Method

(i) Taking logarithms (to base 10) of both sides,

$$\log_{10} y = \log_{10} a + x \log_{10} b$$

(ii) Plot a graph of  $y$  against  $x$  with  $y$  on a logarithmic scale and  $x$  on a linear scale.

(iii) Estimate the the “best straight line”.

(iv) Read off from the graph two sets of co-ordinates,  $(x_1, y_1)$  and  $(x_2, y_2)$ , as far apart as possible.

(v) Solve for  $a$  and  $b$  the simultaneous equations,

$$\log_{10} y_1 = \log_{10} a + x_1 \log_{10} b,$$

$$\log_{10} y_2 = \log_{10} a + x_2 \log_{10} b.$$

If it is possible to choose zero for the  $x_1$  value, so much the better, but this is not essential.

3.

$$y = ae^{bx}.$$

### Method

(i) Taking natural logarithms of both sides,

$$\ln y = \ln a + bx.$$

(ii) Plot a graph of  $y$  against  $x$  with  $y$  on a logarithmic scale and  $x$  on a linear scale.

(iii) Estimate the “best straight line”.

(iv) Read off two sets of co-ordinates,  $(x_1, y_1)$  and  $(x_2, y_2)$ , as far apart as possible.

(v) Solve for  $a$  and  $b$  the simultaneous equations,

$$\ln y_1 = \ln a + bx_1,$$

$$\ln y_2 = \ln a + bx_2.$$

If it possible to choose zero for the  $x_1$  value, so much the better, but this is not essential.