

“JUST THE MATHS”

SLIDES NUMBER

5.10

**GEOMETRY 10
(Graphical solutions)**

by

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5.10.1 Introduction

5.10.2 The graphical solution of linear equations

5.10.3 The graphical solution of quadratic equations

5.10.4 The graphical solution of simultaneous equations

UNIT 5.10 - GEOMETRY 10

GRAPHICAL SOLUTIONS

5.10.1 INTRODUCTION

An algebraic equation in a variable quantity, x , may be written in the general form

$$f(x) = 0$$

where $f(x)$ is an algebraic expression involving x ; we call it a “**function of x** ”; (see Unit 10.1).

In the following, $f(x)$ will be either

(a) a **linear** function of the form $ax + b$, where a and b are constants,

or

(b) a **quadratic** function of the form $ax^2 + bx + c$ where a , b and c are constants.

The solutions of the equation $f(x) = 0$ are those values of x which cause $f(x)$ to take the value zero.

The solutions are also the values of x for which the graph of the equation

$$y = f(x)$$

meets the x -axis.

5.10.2 THE GRAPHICAL SOLUTION OF LINEAR EQUATIONS

To solve the equation

$$ax + b = 0,$$

we may plot the graph of the equation $y = ax + b$ to find the point at which it meets the x -axis.

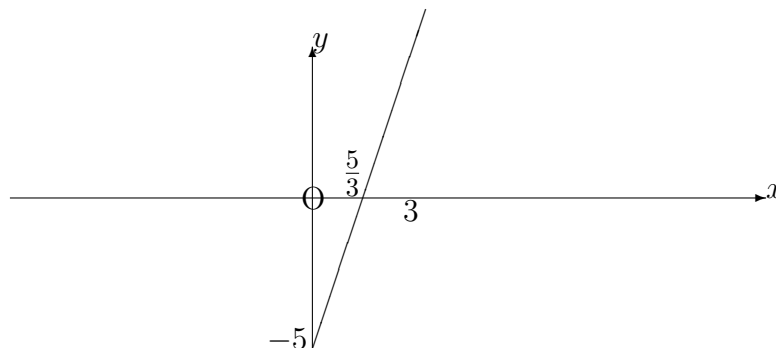
EXAMPLES

1. By plotting the graph of $y = 3x - 5$ from $x = 0$ to $x = 3$, solve the linear equation

$$3x - 5 = 0.$$

Solution

x	0	1	2	3
y	-5	-2	1	4



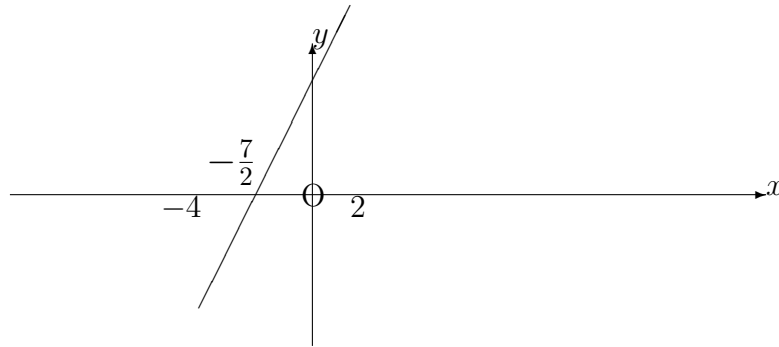
Hence $x \simeq 1.7$

2. By plotting the graph of $y = 2x + 7$ from $x = -4$ to $x = 2$, solve the linear equation

$$2x + 7 = 0.$$

Solution

x	-4	-3	-2	-1	0	1	2
y	-1	1	3	5	7	10	11



Hence $x = -3.5$

5.10.3 THE GRAPHICAL SOLUTION OF QUADRATIC EQUATIONS

To solve the quadratic equation

$$ax^2 + bx + c = 0,$$

we may plot the graph of the equation $y = ax^2 + bx + c$ and determine the points at which it crosses the x -axis.

Alternatively, we may plot the graphs of $y = ax^2 + bx$ and $y = -c$ to determine their points of intersection.

The alternative method is convenient since the first graph passes through the origin.

EXAMPLE

By plotting the graph of $y = x^2 - 4x$ from $x = -2$ to $x = 6$, solve the quadratic equations,

(a)

$$x^2 - 4x = 0;$$

(b)

$$x^2 - 4x + 2 = 0;$$

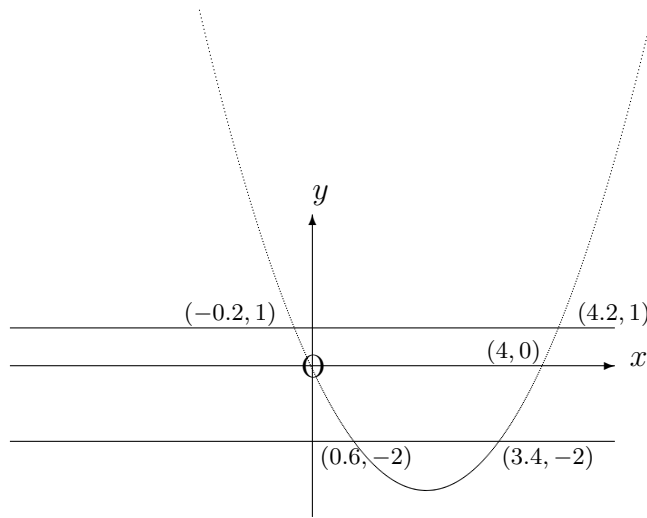
(c)

$$x^2 - 4x - 1 = 0.$$

Solution

x	-2	-1	0	1	2	3	4	5	6
y	12	5	0	-3	-2	-3	0	5	12

For parts (b) and (c), we shall also need the graphs of $y = -2$ and $y = 1$.



Hence the three sets of solutions are:

(a)

$$x = 0 \quad \text{and} \quad x = 4;$$

(b)

$$x \simeq 3.4 \quad \text{and} \quad x \simeq 0.6;$$

(c)

$$x \simeq 4.2 \quad \text{and} \quad x \simeq -0.2$$

5.10.4 THE GRAPHICAL SOLUTION OF SIMULTANEOUS EQUATIONS

Problem

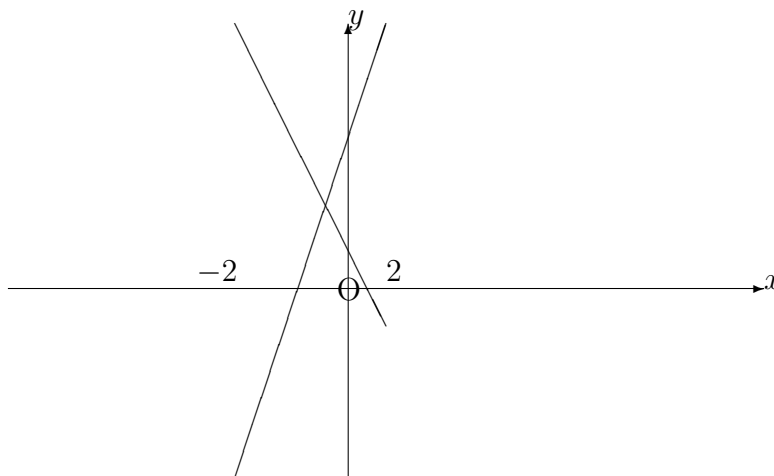
To solve either a pair of simultaneous linear equations or a pair of simultaneous equations consisting of one linear and one quadratic equation.

EXAMPLES

1. By plotting the graphs of $5x + y = 2$ and $-3x + y = 6$ from $x = -2$ to $x = 2$, determine the common solution of the two equations.

Solution

x	-2	-1	0	1	2
$y_1 = 2 - 5x$	12	7	2	-3	-8
$y_2 = 6 + 3x$	0	3	6	9	12



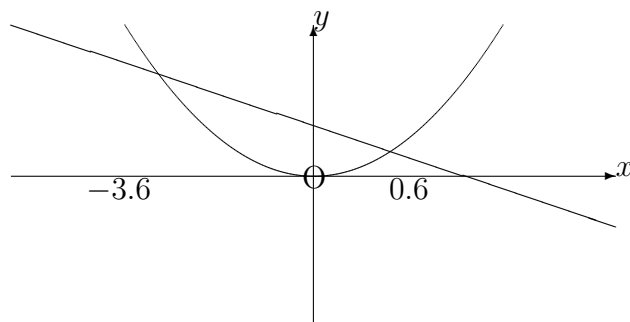
Hence $x = -0.5$ and $y = 4.5$

2. By plotting the graphs of the equations $y = x^2$ and $y = 2 - 3x$ from $x = -4$ to $x = 2$, determine their common solutions and hence solve the quadratic equation

$$x^2 + 3x - 2 = 0.$$

Solution

x	-4	-3	-2	-1	0	1	2
$y_1 = x^2$	16	9	4	1	0	1	4
$y_2 = 2 - 3x$	14	11	8	5	2	-1	-4



Hence $x \simeq 0.6$ and $x \simeq -3.6$