

“JUST THE MATHS”

SLIDES NUMBER

3.3

TRIGONOMETRY 3

(Approximations & inverse functions)

by

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3.3.1 Approximations for trigonometric functions

3.3.2 Inverse trigonometric functions

UNIT 3.3 - TRIGONOMETRY 3

APPROXIMATIONS AND INVERSE FUNCTIONS

3.3.1 APPROXIMATIONS FOR TRIGONOMETRIC FUNCTIONS

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} \dots$$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} \dots$$

$$\tan \theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$$

N.B. θ must be in radians.

If θ is small

$$\sin \theta \simeq \theta;$$

$$\cos \theta \simeq 1;$$

$$\tan \theta \simeq \theta.$$

Better approximations using more terms of the infinite series.

EXAMPLE

Assuming θ^n is negligible when $n > 4$,

$$\begin{aligned}5 + 2 \cos \theta - 7 \sin \theta &\simeq 5 + 2 - \theta^2 + \frac{\theta^4}{12} - 7\theta + 7\frac{\theta^3}{6} \\ &= \frac{1}{12} [\theta^4 + 14\theta^3 - 12\theta^2 - 84\theta + 84].\end{aligned}$$

3.3.2 INVERSE TRIGONOMETRIC FUNCTIONS

(a)

$$\text{Sin}^{-1}x$$

denotes any angle whose sine value is the number x .

It is necessary that $-1 \leq x \leq 1$.

(b)

$$\text{Cos}^{-1}x$$

denotes any angle whose cosine value is the number x .

It is necessary that $-1 \leq x \leq 1$.

(c)

$$\text{Tan}^{-1}x$$

denotes any angle whose tangent value is x .

x may be any value.

Note:

There will be two **basic** values of an inverse function from two different quadrants.

Either value may be increased or decreased by a whole multiple of 360° (2π).

EXAMPLES

1. $\text{Sin}^{-1}(\frac{1}{2}) = 30^\circ \pm n360^\circ$ or $150^\circ \pm n360^\circ$.
2. $\text{Tan}^{-1}(\sqrt{3}) = 60^\circ \pm n360^\circ$ or $240^\circ \pm n360^\circ$.
Alternatively, $\text{Tan}^{-1}(\sqrt{3}) = 60^\circ \pm n180^\circ$.

Another Type of Question

3. Obtain all of the solutions to the equation

$$\cos 3x = -0.432$$

which lie in the interval $-180^\circ \leq x \leq 180^\circ$.

Solution

$3x$ is any one of the angles (within an interval $-540^\circ \leq 3x \leq 540^\circ$) whose cosine is equal to -0.432 .

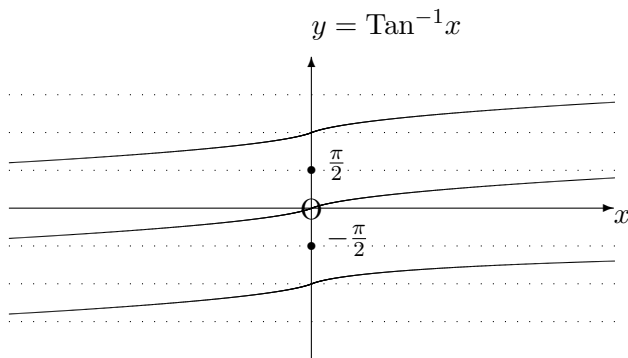
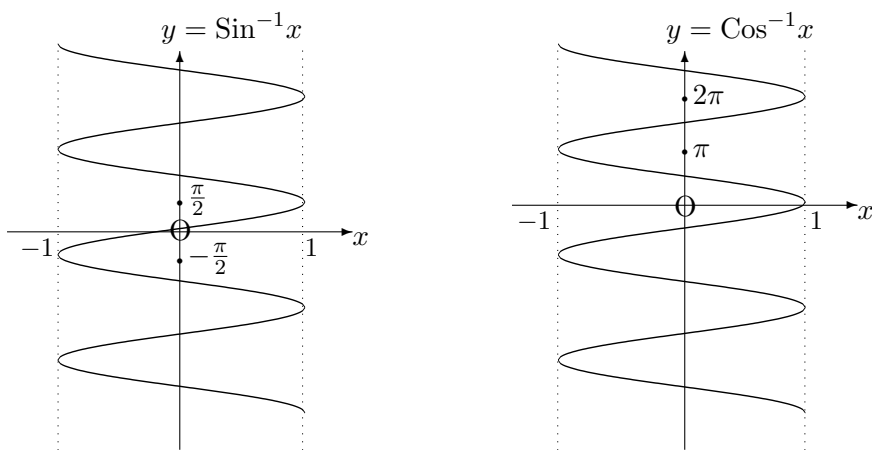
By calculator, the simplest angle is 115.59°

The complete set is

$$\pm 115.59^\circ \quad \pm 244.41^\circ \quad \pm 475.59^\circ$$

$$\text{giving } x = \pm 38.5^\circ \quad \pm 81.5^\circ \quad \pm 158.5^\circ$$

Note: The graphs of inverse trigonometric functions are discussed fully in Unit 10.6, but we include them here for the sake of completeness.



PRINCIPAL VALUE.

This is the unique value which lies in a specified range.

Principal values use the lower-case initial letter of each inverse function.

(a) $\theta = \sin^{-1}x$ lies in the range $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

(b) $\theta = \cos^{-1}x$ lies in the range $0 \leq \theta \leq \pi$.

(c) $\theta = \tan^{-1}x$ lies in the range $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

EXAMPLES

1. $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ or $\frac{\pi}{6}$.

2. $\tan^{-1}(-\sqrt{3}) = -60^\circ$ or $-\frac{\pi}{3}$.

3. Obtain u in terms of v when $v = 5 \cos(1 - 7u)$.

Solution

$$\frac{v}{5} = \cos(1 - 7u);$$

$$\text{Cos}^{-1}\left(\frac{v}{5}\right) = 1 - 7u;$$

$$\text{Cos}^{-1}\left(\frac{v}{5}\right) - 1 = -7u;$$

$$u = -\frac{1}{7} \left[\text{Cos}^{-1}\left(\frac{v}{5}\right) - 1 \right].$$