"JUST THE MATHS"

SLIDES NUMBER

3.2

TRIGONOMETRY 2
(Graphs of trigonometric functions)

by

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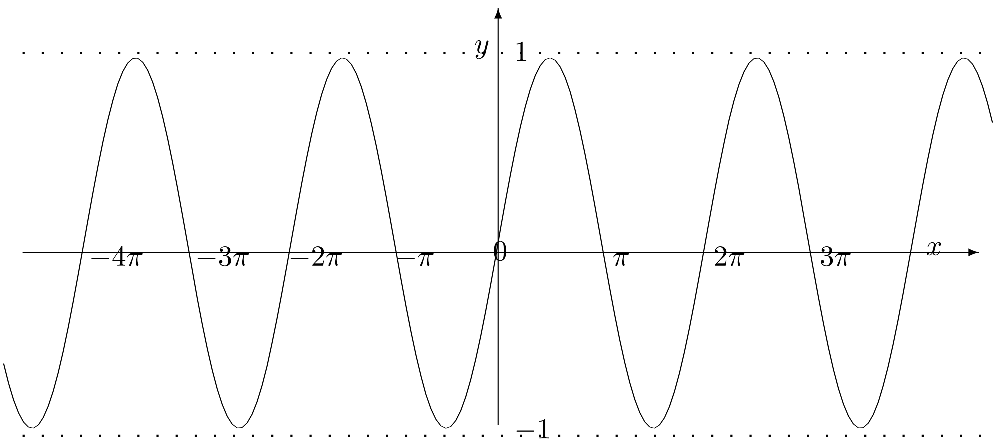
UNIT 3.2 - TRIGONOMETRY 2
GRAPHS OF TRIGONOMETRIC FUNCTIONS

3.2.1 GRAPHS OF ELEMENTARY TRIGONOMETRIC FUNCTIONS

1. $y = \sin\theta$



Results and Definitions

(i)

$$\sin(\theta + 2\pi) \equiv \sin\theta.$$

$\sin\theta$ is a **"periodic function with period 2π"**.

(ii) Other periods are $\pm 2n\pi$ where n is any integer.

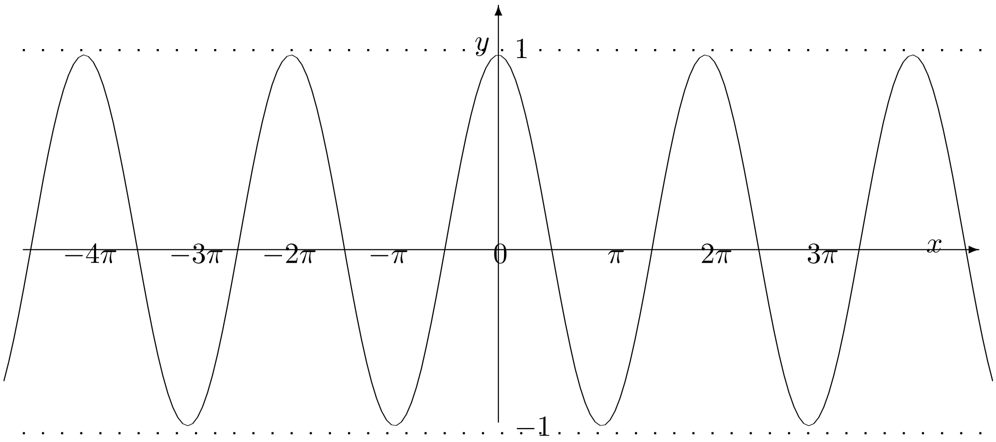
(iii) The smallest positive period is called the **"primitive period"** or **"wavelength"**.

(iv)

$$\sin(-\theta) \equiv -\sin\theta$$

and $\sin\theta$ is called an **"odd function"**.

2. $y = \cos\theta$



Results and Definitions

(i)

$$\cos(\theta + 2\pi) \equiv \cos\theta.$$

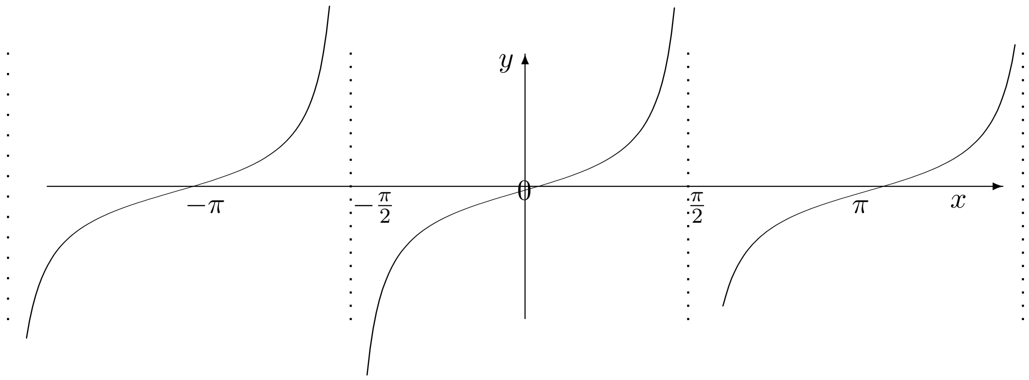
$\cos\theta$ is a periodic function with primitive period 2π.

(ii)

$$\cos(-\theta) \equiv \cos\theta$$

and $\cos\theta$ is called an **“even function”**.

3. $y = \tan\theta$



Results and Definitions

(i)

$$\tan(\theta + \pi) \equiv \tan\theta.$$

$\tan\theta$ is a periodic function with primitive period π.

(ii)

$$\tan(-\theta) \equiv -\tan\theta$$

and $\tan\theta$ is called an **“odd function”**.

3.2.2 GRAPHS OF MORE GENERAL TRIGONOMETRIC FUNCTIONS

In scientific work, it is possible to encounter functions of the form

$$\boxed{\text{Asin}(\omega\theta + \alpha)} \text{ and } \boxed{\text{Acos}(\omega\theta + \alpha)},$$

where ω and α are constants.

EXAMPLES

1. Sketch the graph of

$$y = 5\cos(\theta - \pi).$$

Solution

(i) the graph will have the same shape as the basic cosine wave;

(ii) the graph will lie between $y = -5$ and $y = 5$ so has an **"amplitude"** of 5;

(iii) the graph will cross the θ axis at the points for which

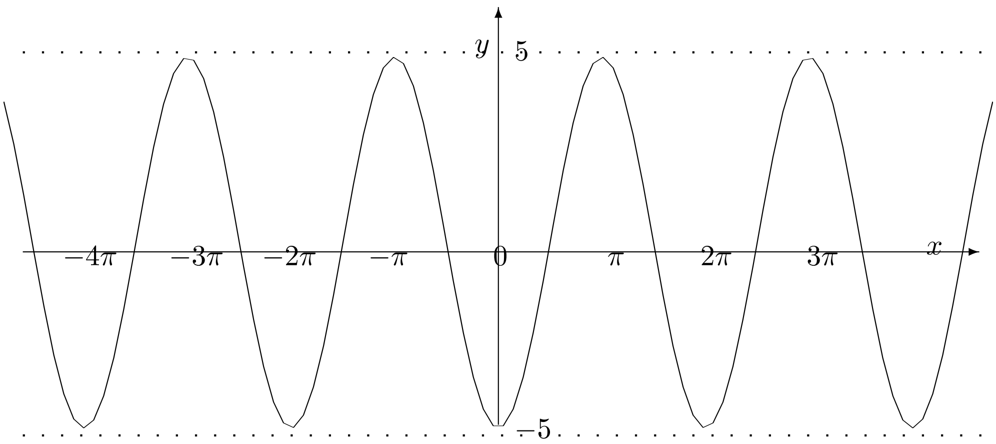
$$\theta - \pi = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, ...$$

that is

$$\theta = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$$

(iv) The y-axis must be placed between the smallest **negative** intersection with the θ-axis and the smallest **positive** intersection with the θ-axis (in proportion to their values).

In this case, the y-axis must be placed half way between $\theta = -\frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$.



Note:

$$5\cos(\theta - \pi) \equiv -5\cos\theta$$

so that graph is an "upsidedown" cosine wave with an amplitude of 5.

Not all examples can be solved in this way.

2. Sketch the graph of

$$y = 3\sin(2\theta + 1).$$

Solution

(i) the graph will have the same shape as the basic sine wave;

(ii) the graph will have an amplitude of 3;

(iii) The graph will cross the θ-axis at the points for which

$$2\theta + 1 = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, ...$$

That is, $\theta =$

$$...-6.78, -5.21, -3.64, -2.07, -0.5, 1.07, 2.64, 4.21, 5.78...$$

(iv) The y-axis must be placed between $\theta = -0.5$ and $\theta = 1.07$ but at about one third of the way from $\theta = -0.5$