"JUST THE MATHS"

SLIDES NUMBER

3.1

TRIGONOMETRY 1
(Angles & trigonometric functions)

by

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UNIT 3.1 - TRIGONOMETRY 1 - ANGLES AND TRIGONOMETRIC FUNCTIONS

3.1.1 INTRODUCTION

The following results will be assumed without proof:

(i) The Circumference, C, and Diameter, D, of a circle are directly proportional to each other through the formula

$$C = \pi D$$

or, if the radius is r,

$$C = 2\pi r.$$

(ii) The area, A, of a circle is related to the radius, r, by means of the formula

$$A = \pi r^2.$$

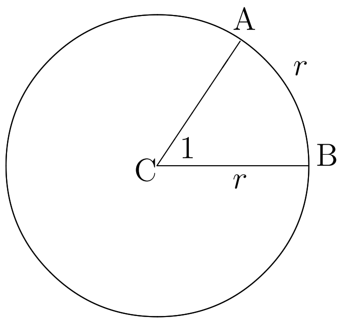
3.1.2 ANGULAR MEASURE

(a) Astronomical Units

The **"degree"** is a $\frac{1}{360}$th part of one complete revolution. It is based on the study of planetary motion where 360 is approximately the number of days in a year.

(b) Radian Measure

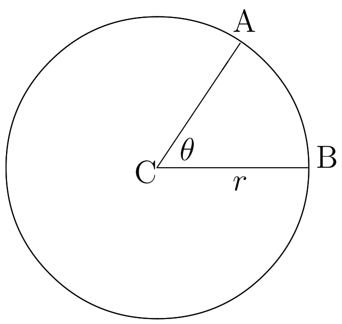
A **"radian"** is the angle subtended at the centre of a circle by an arc which is equal in length to the radius.



RESULTS

(i) There are 2π radians in one complete revolution;

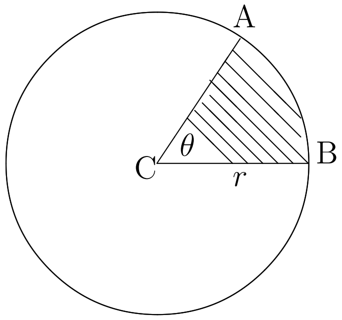
or π radians is equivalent to 180°



(ii) In the above diagram, the arclength from A to B will be given by

$$\frac{\theta}{2\pi} \times 2\pi r = r\theta,$$

assuming that θ is measured in radians.



(iii) In the above diagram, the area of the sector ABC is given by

$$\frac{\theta}{2\pi} \times \pi r^2 = \frac{1}{2} r^2 \theta.$$

(c) Standard Angles

The scaling factor for converting degrees to radians is

$$\frac{\pi}{180}$$

and the scaling factor for converting from radians to degrees is

$$\frac{180}{\pi}.$$

ILLUSTRATIONS

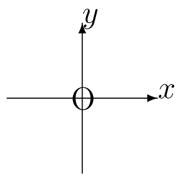
1. 15° is equivalent to $\frac{\pi}{180} \times 15 = \frac{\pi}{12}$.
2. 30° is equivalent to $\frac{\pi}{180} \times 30 = \frac{\pi}{6}$.
3. 45° is equivalent to $\frac{\pi}{180} \times 45 = \frac{\pi}{4}$.
4. 60° is equivalent to $\frac{\pi}{180} \times 60 = \frac{\pi}{3}$.

5. 75° is equivalent to $\frac{\pi}{180} \times 75 = \frac{5\pi}{12}$.

6. 90° is equivalent to $\frac{\pi}{180} \times 90 = \frac{\pi}{2}$.

(d) Positive and Negative Angles

Using cartesian axes Ox and Oy, the **"first quadrant"** is that for which x and y are both positive, and the other three quadrants are numbered from the first in an anti-clockwise sense.



From the positive x-direction, we measure angles positively in the anticlockwise sense and negatively in the clockwise sense. Special names are given the to type of angles obtained as follows:

1. Angles in the range between 0° and 90° are called **"positive acute"** angles

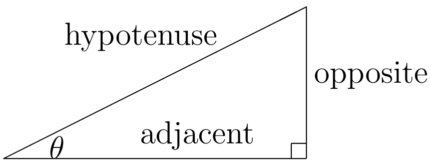
2. Angles in the range between 90° and 180° are called **"positive obtuse"** angles.

3. Angles in the range between 180° and 360° are called **"positive reflex"** angles.

4. Angles measured in the clockwise sense have similar names but preceded by the word **"negative"**.

3.1.3 TRIGONOMETRIC FUNCTIONS

For future reference, we shall assume, without proof, the result known as **"Pythagoras' Theorem"**. This states that the square of the length of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the lengths of the other two sides.



DEFINITIONS

(a) **"Sine"**

$$\sin\theta \equiv \frac{\text{opposite}}{\text{hypotenuse}};$$

(b) **"Cosine"**

$$\cos\theta \equiv \frac{\text{adjacent}}{\text{hypotenuse}};$$

(c) **"Tangent"**

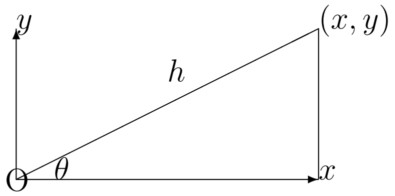
$$\tan\theta \equiv \frac{\text{opposite}}{\text{adjacent}}.$$

Notes:

(i) To remember the above, use

S.O.H.C.A.H.T.O.A.

(ii) The definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$ can be extended to angles of any size:



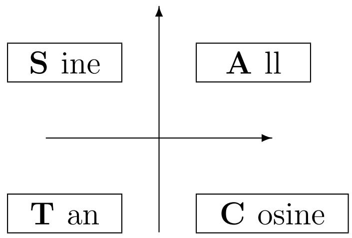
$$\sin\theta \equiv \frac{y}{h};$$

$$\cos\theta \equiv \frac{x}{h};$$

$$\tan\theta \equiv \frac{y}{x} \equiv \frac{\sin\theta}{\cos\theta}.$$

Trigonometric functions can also be called **"trigonometric ratios"**.

(iii) Basic trigonometric functions have positive values in the following quadrants.



(iv) Three other trigonometric functions are sometimes used and are defined as the reciprocals of the three basic functions as follows:

"Secant"

$$\sec\theta \equiv \frac{1}{\cos\theta};$$

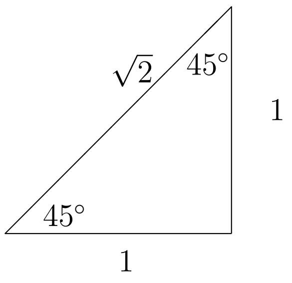
"Cosecant"

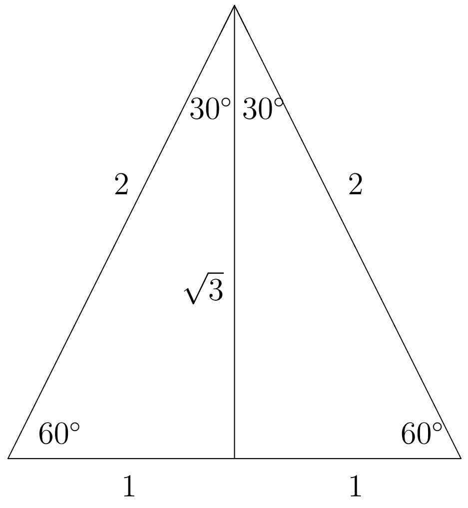
$$\mathrm{cosec}\theta \equiv \frac{1}{\sin\theta};$$

"Cotangent"

$$\cot\theta \equiv \frac{1}{\tan\theta}.$$

(v) The values of the functions $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the particular angles 30°, 45° and 60° are easily obtained without calculator from the following diagrams:





The diagrams show that

(a) $\sin 45° = \frac{1}{\sqrt{2}}$; (b) $\cos 45° = \frac{1}{\sqrt{2}}$; (c) $\tan 45° = 1$;

(d) $\sin 30° = \frac{1}{2}$; (e) $\cos 30° = \frac{\sqrt{3}}{2}$; (f) $\tan 30° = \frac{1}{\sqrt{3}}$;

(g) $\sin 60° = \frac{\sqrt{3}}{2}$; (h) $\cos 60° = \frac{1}{2}$; (i) $\tan 60° = \sqrt{3}$.