"JUST THE MATHS"

SLIDES NUMBER

13.2

INTEGRATION APPLICATIONS 2
(Mean values)
&
(Root mean square values)

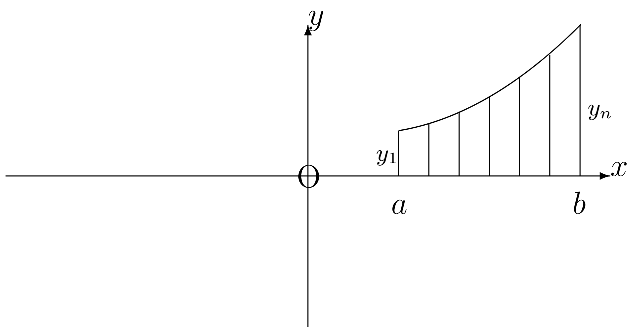
by

A.J.Hobson

UNIT 13.2 - INTEGRATION APPLICATIONS 2

MEAN & ROOT MEAN SQUARE VALUES

13.2.1 MEAN VALUES



On the curve whose equation is

$$y = f(x),$$

let $y_1, y_2, y_3, ..., y_n$ be the y-coordinates at n different x-coordinates,

$$a = x_1, x_2, x_3, ..., x_n = b.$$

The average (that is, the arithmetic mean) of these n y-coordinates is

$$\frac{y_1 + y_2 + y_3 + ... + y_n}{n}.$$

The problem is to determine the average (arithmetic mean) of **all** the y-coordinates, from $x = a$ to $x = b$ on the curve whose equation is $y = f(x)$.

We take a very **large** number, n, of y-coordinates separated in the x-direction by very **small** distances.

If these distances are typically represented by δx then the required mean value could be written

$$\frac{y_1\delta x + y_2\delta x + y_3\delta x + ... + y_n\delta x}{n\delta x}.$$

The denominator is equivalent to $(b-a+\delta x)$, since there are only $n-1$ spaces between the n y-coordinates.

Allowing the number of y-coordinates to increase indefinitely, δx will to tend to zero.

Hence, the **"Mean Value"** is given by

$$\text{M.V.} = \frac{1}{b-a} \lim_{\delta x \to 0} \sum_{x=a}^{x=b} y\delta x.$$

That is,

$$\text{M.V.} = \frac{1}{b-a} \int_a^b f(x) \, \mathrm{d}x.$$

Note:
The Mean Value provides the height of a rectangle, with base $b - a$, having the same area as the net area between the curve and the x-axis.

EXAMPLE

Determine the mean value of the function,

$$f(x) \equiv x^2 - 5x,$$

from $x = 1$ to $x = 4$.

Solution

The Mean Value is given by

$$\text{M.V.} = \frac{1}{4-1}\int_1^4 \ (x^2 - 5x)\ \mathrm{d}x$$

$$= \frac{1}{3}\left[\frac{x^3}{3} - \frac{5x^2}{2}\right]_1^4$$

$$= \frac{1}{3}\left[\left(\frac{64}{3} - 40\right) - \left(\frac{1}{3} - \frac{5}{2}\right)\right] = -\frac{33}{2}.$$

13.2.2 ROOT MEAN SQUARE VALUES

It is sometimes convenient to use an alternative kind of average for the values of a function, $f(x)$, between $x = a$ and $x = b$

The **"Root Mean Square Value"** provides a measure of "central tendency" for the **numerical** values of $f(x)$.

The Root Mean Square Value is defined to be the square root of the mean value of $f(x)$ from $x = a$ to $x = b$.

$$\text{R.M.S.V.} = \sqrt{\frac{1}{b-a}\int_a^b \ [f(x)]^2 \ \mathrm{d}x}.$$

EXAMPLE

Determine the Root Mean Square Value of the function,

$$f(x) \equiv x^2 - 5,$$

from $x = 1$ to $x = 3$.

Solution

The Root Mean Square Value is given by

$$\text{R.M.S.V.} = \sqrt{\frac{1}{3-1}\int_1^3 \ (x^2-5)^2 \ \mathrm{d}x}.$$

First, we determine the **"Mean Square Value"**.

$$\text{M.S.V.} = \frac{1}{2}\int_1^3 \ (x^4 - 10x^2 + 25) \ \mathrm{d}x.$$

$$= \frac{1}{2}\left[\frac{x^5}{5} - \frac{10x^3}{3} + 25x\right]_1^3$$

$$= \frac{1}{2}\left[\left(\frac{243}{5} - \frac{270}{3} + 75\right) - \left(\frac{1}{5} - \frac{10}{3} + 25\right)\right]$$

$$= \frac{1}{30}\left[(729 - 1350 + 1125) - (3 - 50 + 375)\right]$$

$$= \frac{176}{30}.$$

Thus,

$$\text{R.M.S.V.} = \sqrt{\frac{176}{30}} \simeq 2.422$$