"JUST THE MATHS"

SLIDES NUMBER

12.2

INTEGRATION 2
(Introduction to definite integrals)

by

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12.2.1 Definition and examples

UNIT 12.2 - INTEGRATION 2
INTRODUCTION TO DEFINITE INTEGRALS

12.2.1 DEFINITION AND EXAMPLES

In Unit 12.1, all the integrals were **"indefinite integrals"**.

Each result contained an arbitrary constant which cannot be assigned a value without further information.

In practical applications , we encounter **"definite integrals"**, which are represented by a numerical value.

DEFINITION

Suppose that

$$\int f(x)\mathrm{d}x = g(x) + C.$$

Then the symbol

$$\int_a^b f(x)\mathrm{d}x$$

is used to mean

(Value of $g(x) + C$ at $x = b$)
minus
(Value of $g(x) + C$ at $x = a$).

C will cancel out; hence,

$$\int_a^b f(x)\mathrm{d}x = g(b) - g(a).$$

The right hand side can also be written

$$[g(x)]_a^b.$$

a is the **"lower limit"** of the definite integral.

b is the **"upper limit"** of the definite integral.

EXAMPLES

1. Evaluate the definite integral

$$\int_0^{\frac{\pi}{2}} \cos x\mathrm{d}x.$$

Solution

$$\int_0^{\frac{\pi}{2}} \cos x\mathrm{d}x = [\sin x]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1.$$

2. Evaluate the definite integral

$$\int_1^3 (2x+1)^2\mathrm{d}x.$$

Solution

$$\int_1^3 (2x+1)^2\mathrm{d}x = \left[\frac{(2x+1)^3}{6}\right]_1^3 = \frac{7^3}{6} - \frac{3^3}{6} \simeq 52.67$$

Notes:

(i) Alternatively,

$$\int_1^3 (2x+1)^2 \, \mathrm{d}x = \int_1^3 \left(4x^2 + 4x + 1\right) \, \mathrm{d}x$$

$$= \left[4\frac{x^3}{3} + 2x^2 + x\right]_1^3 .$$

The expression in the brackets differs only from the previous result by the constant value $\frac{1}{6}$.

Hence the numerical result for the definite integral will be the same.

(ii) Another alternative method is to substitute $u = 2x+1$; but the limits of integration should be changed to the appropriate values for u.

Replace $\mathrm{d}x$ by $\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ (that is, $\frac{1}{2}\mathrm{d}u$).

Replace $x = 1$ and $x = 3$ by $u = 2 \times 1 + 1 = 3$ and $u = 2 \times 3 + 1 = 7$, respectively.

We obtain

$$\int_3^7 u^2 \frac{1}{2} \mathrm{d}u = \left[\frac{u^3}{6}\right]_3^7 = \frac{7^3}{6} - \frac{3^3}{6} \simeq 52.67$$