# “JUST THE MATHS”

# SLIDES NUMBER

# 1.7

# ALGEBRA 7
# (Simultaneous linear equations)

## by


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# UNIT 1.7 - ALGEBRA 7
# SIMULTANEOUS LINEAR EQUATIONS

## 1.7.1 TWO SIMULTANEOUS LINEAR EQUATIONS IN TWO UNKNOWNS

$$\begin{aligned} ax + by &= p, \\ cx + dy &= q. \end{aligned}$$

First eliminate one of the variables (eg. $x$) in order to calculate the other.

$$\begin{aligned} cax + cby &= cp, \\ acx + ady &= aq. \end{aligned}$$

$$y(cb - ad) = cp - aq;$$

$$y = \frac{cp - aq}{cb - ad} \ \text{ if } \ cb - ad \neq 0.$$

To find $x$, substitute back or eliminate $y$.

**Degenerate Case.**
If $cb - ad = 0$, the left hand sides of the two equations are proportional to each other.

## EXAMPLE

Solve the simultaneous linear equations

$$\begin{aligned} 6x - 2y &= 1, \qquad (1) \\ 4x + 7y &= 9. \qquad (2) \end{aligned}$$

$$\begin{aligned} 24x - 8y &= 4, \qquad (4) \\ 24x + 42y &= 54. \qquad (5) \end{aligned}$$

Hence, $-50y = -50$ and $y = 1$.

Substituting into (1), $6x - 2 = 1$ giving $6x = 3$.

Hence, $x = \frac{1}{2}$.

## Alternative Method

$$\begin{aligned} 42x - 14y &= 7, \qquad (5) \\ -8x - 14y &= -18. \qquad (6) \end{aligned}$$

Hence, $50x = 25$, so $x = \frac{1}{2}$.

Substituting into (1) gives $3 - 2y = 1$, so $y = 1$.

## 1.7.2 THREE SIMULTANEOUS LINEAR EQUATIONS IN THREE UNKNOWNS

$$\begin{aligned} a_1x + b_1y + c_1z &= k_1, \\ a_2x + b_2y + c_2z &= k_2, \\ a_3x + b_3y + c_3z &= k_3. \end{aligned}$$

Eliminate one of the variables from two different pairs of the three equations.

## EXAMPLE

Solve, for $x$, $y$ and $z$, the simultaneous linear equations

$$\begin{aligned} x - y + 2z &= 9, & (1) \\ 2x + y - z &= 1, & (2) \\ 3x - 2y + z &= 8. & (3) \end{aligned}$$

## Solution

Eliminating $z$ from equations (2) and (3),

$$5x - y = 9. \qquad (4)$$

Eliminating $z$ from equations (1) and (2),

$$5x + y = 11. \qquad (5)$$

Adding (4) to (5),

$$10x = 20 \quad \text{or} \quad x = 2.$$

Subtracting (4) from (5),

$$2y = 2 \quad \text{or} \quad y = 1.$$

Substituting $x$ and $y$ into (3),

$$z = 8 - 3x + 2y = 8 - 6 + 2 = 4$$

Thus,

$$x = 2, \quad y = 1 \quad \text{and} \quad z = 4.$$

### 1.7.3 ILL-CONDITIONED EQUATIONS

Rounding errors may swamp the values of the variables being solved for.

**EXAMPLE**

$$\begin{aligned} x + y &= 1, \\ 1.001x + y &= 2 \end{aligned}$$

have the common solution $x = 1000$, $y = -999$.

$$\begin{aligned} x + y &= 1, \\ x + y &= 2 \end{aligned}$$

have no solution at all.

$$\begin{aligned} x + y &= 1, \\ 0.999x + y &= 2 \end{aligned}$$

have solutions $x = -1000$, $y = 1001$.