“JUST THE MATHS”

SLIDES NUMBER

1.6

ALGEBRA 6
(Formulae and algebraic equations)

by

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UNIT 1.6 - ALGEBRA 6
FORMULAE AND ALGEBRAIC EQUATIONS

1.6.1 TRANSPOSITION OF FORMULAE

The following steps may be carried out on both sides of a given formula:

(a) Addition or subtraction of the same value;

(b) Multiplication or division by the same value;

(c) The raising of both sides to equal powers;

(d) Taking logarithms of both sides.

EXAMPLES

1. Make x the subject of the formula

$$y = 3(x + 7).$$

Solution

$$\frac{y}{3} = x + 7;$$

$$x = \frac{y}{3} - 7.$$

2. Make y the subject of the formula

$$a = b + c\sqrt{x^2 - y^2}.$$

Solution

$$a - b = c\sqrt{x^2 - y^2};$$

$$\frac{a-b}{c} = \sqrt{x^2 - y^2};$$

$$\left(\frac{a-b}{c}\right)^2 = x^2 - y^2;$$

$$\left(\frac{a-b}{c}\right)^2 - x^2 = -y^2;$$

$$x^2 - \left(\frac{a-b}{c}\right)^2 = y^2;$$

$$y = \pm\sqrt{x^2 - \left(\frac{a-b}{c}\right)^2}.$$

3. Make x the subject of the formula

$$e^{2x-1} = y^3.$$

Solution

Taking natural logarithms of both sides of the formula

$$2x - 1 = 3\ln y.$$

Hence

$$x = \frac{3\ln y + 1}{2}.$$

Note:

For scientific formulae, ignore the negative square roots.

1.6.2 SOLUTION OF LINEAR EQUATIONS

If $ax + b = c$, then $x = \frac{c-b}{a}$.

EXAMPLES

1. Solve the equation

$$5x + 11 = 20.$$

$$\text{Ans : } \; x = \frac{20 - 11}{5} = \frac{9}{5} = 1.8$$

2. Solve the equation

$$3 - 7x = 12.$$

$$\text{Ans}: \;\; x = \frac{12-3}{-7} = \frac{9}{-7} \simeq -1.29$$

1.6.3 SOLUTION OF QUADRATIC EQUATIONS

Standard form is $ax^2 + bx + c = 0$.

(a) By Factorisation

EXAMPLES

1. Solve the quadratic equation

$$6x^2 + x - 2 = 0.$$

   In factorised form,

$$(3x+2)(2x-1) = 0.$$

   Hence, $x = -\frac{2}{3}$ or $x = \frac{1}{2}$.

2. Solve the quadratic equation

$$15x^2 - 17x - 4 = 0.$$

   In factorised form

$$(5x+1)(3x-4) = 0.$$

   Hence, $x = -\frac{1}{5}$ or $x = \frac{4}{3}$.

(b) By Completing the square

EXAMPLES

1. Solve the quadratic equation

$$x^2 - 4x - 1 = 0.$$

Equation can be written

$$(x-2)^2 - 5 = 0.$$

Thus,

$$x - 2 = \pm\sqrt{5}.$$

$$\text{Ans}: \quad x = 2 \pm \sqrt{5} \simeq 4.236 \;\; \text{or} \quad -0.236$$

2. Solve the quadratic equation

$$4x^2 + 4x - 2 = 0.$$

Equation can be written

$$4\left[x^2 + x - \frac{1}{2}\right] = 0;$$

$$4\left[\left(x + \frac{1}{2}\right)^2 - \frac{3}{4}\right] = 0.$$

Hence,

$$\left(x + \frac{1}{2}\right)^2 = \frac{3}{4};$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{3}{4}};$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{3}{4}} \text{ or } \frac{-1 \pm \sqrt{3}}{2}.$$

(c) By the Quadratic Formula

Given

$$ax^2 + bx + c = 0,$$

$$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = 0;$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0;$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}};$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}};$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Note:

$b^2 - 4ac$ is called the **"Discriminant"**.

The discriminant gives two solutions, one solution (coincident pair) or no (real) solutions according as its value is positive, zero or negative.

EXAMPLES

Use the quadratic formula to solve the following:

1.

$$2x^2 - 3x - 7 = 0.$$

Solution

$$x = \frac{3 \pm \sqrt{9+56}}{4} = \frac{3 \pm \sqrt{65}}{4}$$

$$= \frac{3 \pm 8.062}{4} \simeq 2.766 \quad \text{or} \quad -1.266$$

2.

$$9x^2 - 6x + 1 = 0.$$

$$x = \frac{6 \pm \sqrt{36-36}}{18} = \frac{6}{18} = \frac{1}{3} \quad \text{only.}$$

3.

$$5x^2 + x + 1 = 0.$$

Solution

$$x = \frac{-1 \pm \sqrt{1-20}}{10}.$$

No real solutions.