

“JUST THE MATHS”

SLIDES NUMBER

1.10

**ALGEBRA 10
(Inequalities 1)**

by

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1.10.1 Introduction
1.10.2 Algebraic rules for inequalities
1.10.3 Intervals

UNIT 1.10 - ALGEBRA 10

INEQUALITIES 1.

1.10.1 INTRODUCTION

$a < b$ means “a is less than b”.

$b > a$ means “b is greater than a”.

These are “**strict inequalities**”.

eg. if a is the number of days in a particular month and b is the number of hours in that month, then $b > a$.

“**Weak inequalities**” are written in one of the forms

$$a \leq b \quad b \leq a \quad a \geq b \quad b \geq a.$$

eg. if a is the number of students who enrolled for a particular module in a university and b is the number of students who eventually passed that module, then $a \geq b$.

1.10.2 ALGEBRAIC RULES FOR INEQUALITIES

Suppose $a < b$; then,

1. $a + c < b + c$ for any number c .
2. $ac < bc$ when c is positive but $ac > bc$ when c is negative.
3. $\frac{1}{a} > \frac{1}{b}$ provided a and b are **both positive**.

Note:

If a is negative and b is positive, $\frac{1}{a} < \frac{1}{b}$

EXAMPLES

1. Simplify the inequality

$$2x + 3y > 5x - y + 7.$$

Solution

$$-3x + 4y > 7 \quad \text{or} \quad 3x - 4y + 7 < 0$$

2. Solve the inequality

$$\frac{1}{x-1} < 2$$

assuming that $x \neq 1$.

Solution

(a) If $x > 1$, i.e. $x - 1$ is positive,

$$1 < 2(x - 1) \quad \text{or} \quad x - 1 > \frac{1}{2}.$$

Hence,

$$x > \frac{3}{2}.$$

(b) If $x < 1$, i.e. $x - 1$ is negative,

Any negative number is bound to be less than a positive number.

Hence inequality always true when $x < 1$.

Conclusion:

$$x < 1 \text{ and } x > \frac{3}{2}.$$

3. Solve the inequality

$$2x - 7 \leq 3.$$

Solution

Adding 7, then dividing by 2 gives $x \leq 5$.

4. Solve the inequality

$$\frac{x - 1}{x - 6} \geq 0$$

Solution

Left hand side equals zero only when $x = 1$.

(a) If $x - 1 > 0$ and $x - 6 > 0$, then $x > 6$.

(b) If $x - 1 < 0$ and $x - 6 < 0$; then $x < 6$.

Note:

As x passes through 6 there is a sudden change from $-\infty$ to $+\infty$.

Conclusion:

The inequality is satisfied when either $x > 6$ or $x \leq 1$.

1.10.3 INTERVALS

(a) $a < x < b$ denotes an “**open interval**” of all the values of x between a and b but excluding a and b themselves.

(a, b) also means the same thing.

eg. if x is a purely decimal quantity, $-1 < x < 1$.

(b) $a \leq x \leq b$ denotes a “**closed interval**” of all the values of x from a to b inclusive.

$[a, b]$ also means the same thing.

eg. $\sqrt{1 - x^2}$ has real values only when $-1 \leq x \leq 1$.

Note: $a < x \leq b$ or $a \leq x < b$ i.e. $(a, b]$ and $[a, b)$ are “**half-open**” or “**half-closed**”.

(c) $x > a$ $x \geq a$ $x < a$ $x \leq a$ are called “**infinite intervals**”.