"JUST THE MATHS"

SLIDES NUMBER

1.1

ALGEBRA 1
(Introduction to algebra)

by

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UNIT 1.1 - ALGEBRA 1
INTRODUCTION TO ALGEBRA

DEFINITION

An "Algebra" uses Equality (=), Addition (+), Subtraction (-), Multiplication ($\times$ or .) and Division ($\div$).

The Algebra of Numbers = "ARITHMETIC"

1.1.1 THE LANGUAGE OF ALGEBRA

a, b and c denote **constant** numbers of arithmetic;
x, y and z denote **variable** numbers of arithmetic

(a) $a + b$ is the "sum of a and b".

$a + a$ is written $2a$, $\quad a + a + a$ is written $3a$.

(b) $a - b$ is the "difference of a and b".

(c) $a \times b$, $a.b$, ab is the "product of a and b".

$a.a$ is written a^2 $\quad a.a.a$ is written a^3

$-1 \times a$ is written $-a$ and is the "negation" of a.

(d) $a \div b$ or $\frac{a}{b}$ is the "quotient" or "ratio" of a and b.

(e) $\frac{1}{a}$, [also written a^{-1}], is the "reciprocal" of a.

(f) $\mid a \mid$ is the "modulus", "absolute value" or "numerical value" of a.

$\mid a \mid = a$ when a is positive or zero;
$\mid a \mid = -a$ when a is negative or zero.

Rules for combining fractions.

1. $$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$
2. $$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$
3. $$\frac{a}{b} . \frac{c}{d} = \frac{a.c}{b.d}$$
4. $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a.d}{b.c}$$

EXAMPLES

1. How much more than the difference of 127 and 59 is the sum of 127 and 59 ?

   Difference of 127 and 59 is $127 - 59 = 68$.
   Sum of 127 and 59 is $127 + 59 = 186$.
   Sum exceeds the difference by $186 - 68 = 118$.

2. What is the reciprocal of the number which is 5 multiplied by the difference of 8 and 2 ?

   Reciprocal of $5.(8-2) = \frac{1}{30}$.

3. Calculate the value of $4\frac{2}{3} - 5\frac{1}{9}$ expressing the answer as a fraction.

   $$\frac{14}{3} - \frac{46}{9} = \frac{126-138}{27} = -\frac{12}{27} = -\frac{4}{9} \text{ or } \frac{42}{9} - \frac{46}{9} = -\frac{4}{9}$$

4. Remove the modulus signs from the expression $\mid a-2 \mid$ in the cases when (i) a is greater than (or equal to) 2 and (ii) a is less than 2.

   (i) If a is greater than or equal to 2,

   $$\mid a-2 \mid = a-2$$

   (ii) If a is less than 2,

   $$\mid a-2 \mid = -(a-2) = 2-a$$

1.1.2 THE LAWS OF ALGEBRA

(a) The Commutative Law of Addition

$$a + b = b + a$$

(b) The Associative Law of Addition

$$a + (b + c) = (a + b) + c$$

(c) The Commutative Law of Multiplication

$$a.b = b.a$$

(d) The Associative Law of Multipication

$$a.(b.c) = (a.b).c$$

(e) The Distributive Laws

$$a.(b+c) = a.b + a.c$$

$$(a+b).c = a.c + b.c$$

Note for later: $(a+b).(c+d) = a.c + b.c + a.d + b.d$

1.1.3 PRIORITIES IN CALCULATIONS

Problem: $5 \times 6 - 4 = 30$ - $4 = 26$ or $5 \times 2 = 10$????

B.O.D.M.A.S.

B	brackets	( )	First Priority
O	of	$\times$	Joint Second Priority
D	division	$\div$	Joint Second Priority
M	multiplication	$\times$	Joint Second Priority
A	addition	+	Joint Third Priority
S	subtraction	-	Joint Third Priority

Exs.

$5 \times (6 - 4) = 5 \times 2 = 10$
$5 \times 6 - 4 = 30 - 4 = 26.$

$12 \div 3 - 1 = 4 - 1 = 3$
$12 \div (3 - 1) = 12 \div 2 = 6.$

1.1.4 FACTORS

If a number can be expressed as a product of other numbers, each of those other numbers is called a "Factor" of the original number.

EXAMPLES

1. $$70 = 2 \times 7 \times 5$$

   These are "prime" factors

2. Show that the numbers 78 and 182 have two common factors which are prime numbers.

   $$78 = 2 \times 3 \times 13$$

   $$182 = 2 \times 7 \times 13$$

   Common factors are 2 and 13 - both prime.

Highest Common Factor, h.c.f.

$$90 = 2 \times 3 \times 3 \times 5$$

and

$$108 = 2 \times 2 \times 3 \times 3 \times 3$$

$$\text{h.c.f} = 2 \times 3 \times 3 = 18$$

Lowest Common Multiple, l.c.m.

$$15 = 3 \times 5$$

and

$$20 = 2 \times 2 \times 5$$

$$\text{l.c.m.} = 2 \times 2 \times 3 \times 5 = 60$$

Lowest Terms

Common factors may be cancelled to leave the fraction in its "lowest terms".

$$\frac{15}{105} = \frac{3 \times 5}{3 \times 5 \times 7} = \frac{1}{7}$$